

**MOTION & TRACKING**

**Readings:**

Szeliski: Chapter 8.1,8.2,8.4

Some Slides adapted from Univ. of Washington

Additional material by courtesy of Tomas Svoboda  
svoboda@cmp.felk.cvut.cz

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Why estimate visual motion?

- Visual Motion can be annoying
  - Camera instabilities, jitter
  - Measure it; remove it (stabilize)
- Visual Motion indicates dynamics in the scene
  - Moving objects, behavior
  - Track objects and analyze trajectories
- Visual Motion reveals spatial layout
  - Motion parallax

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TX-Clip

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Motion

- Motion Blur
- Motion Detection
  - Find movement
- Moving Object Detection and Location
  - Motion Based Segmentation
  - Trajectories → Tracking
- Shape from Motion
  - Similar to Stereo
  - Optical Flow

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Motion

Information about *scene motion* rather than the *scene*.

an "image cube"  
 $I(x,y,t)$

time

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Process of images over time.

Image sequence:

- An **image sequence** is a series of  $N$  images, or frames, acquired at discrete time instants  $t_k = t_0 + k\Delta t$ , where  $\Delta t$  is a fixed time interval and  $k = 0, 1, \dots, N-1$

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### Motion Blur

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### Image distortion

$$g(i, j) = s \left[ \iint_{(a,b) \in \Omega} f(a,b)h(a,b,i, j)dad b \right] + v(i, j)$$

assuming linearity

$$g(i, j) = (f * h)(i, j) + v(i, j)$$

f-domain (Fourier transform)

$$G(u, v) = F(u, v)H(u, v) + N(u, v)$$

g(i,j) ... degraded image  
f(a,b) ... original image  
h(...) ... distortion function  
v(i,j) ... noise

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### Inverse filtration

$$F(u, v) = G(u, v)H^{-1}(u, v) - N(u, v)H^{-1}(u, v)$$

Relative motion of camera and object

$$H(u, v) = \frac{\sin(\pi V T u)}{\pi V u}$$

V ... constant speed in x-axes  
T ... shutter open time

Poor results in presence of noise  $\Rightarrow$  Wiener filter

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### Inverse filtration example

Blurred Image      Simple inverse filtration

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### Wiener filtration

Blurred + noise added

Simple inverse filtration      Wiener filtration

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### Motion Detection

- Separating background and foreground
  - static scene (background)
  - moving objects (foreground)
- Detect meaningful motions
- Problems:
  - shadows
  - objects that temporarily stopped
  - moving camera / background

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### Motion Detection with Static Camera

- Frame Differencing:
 
$$d(x, y) = \begin{cases} 0 & |f_t(x, y) - f_{t+1}(x, y)| < \epsilon \\ 1 & \text{sonst} \end{cases}$$



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### Motion Detection with Static Camera

- Frame Differencing (Thresholds)
 

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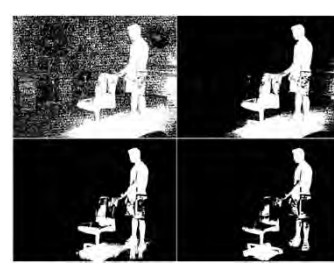
### Motion Detection with Static Camera

- Background Substraction
 
$$d(x, y) = \begin{cases} 0 & |f_t(x, y) - B(x, y)| < \epsilon \\ 1 & \text{sonst} \end{cases}$$



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### Motion Detection with Static Camera

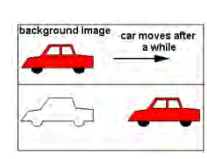
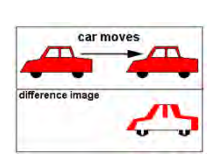
- Frame Differencing (Thresholds)
 

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### Motion Detection Summary

Problems:



- Background subtraction:
  - How to get background model?
  - When to update
  - Holes
- Frame differencing:
  - Parts of object
  - Timescale of frames
  - Threshold

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### Motion Detection

Surveillance:

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**Motion Analysis Problems**

**Correspondence Problem**

- Track corresponding elements across frames

**Reconstruction Problem**

- Given a number of corresponding elements, and camera parameters, what can we say about the 3D motion and structure of the observed scene?

**Segmentation Problem**

- What are the regions of the image plane which correspond to *different* moving objects?

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**Motion field**

Vector field where each vector represents motion of point across image at time interval  $dt$

**Optical Flow**

**Image sequence:**

- Spatial- and Temporal sampling
- Temporal sampling needs to be high
- Correspondence problem (occlusions, etc)

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Assuming that illumination does not change:

Image changes are due to the **RELATIVE MOTION** between the scene and the camera.

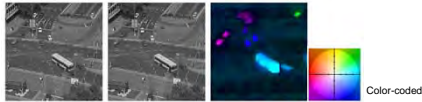
There are 3 possibilities:

- Camera still, moving scene
- Moving camera, still scene
- Moving camera, moving scene

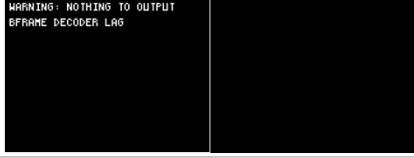
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**Optical Flow Examples**

**Static Camera / Moving Scene**



**Moving Camera / Moving Scene**



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**Motion Field (MF)**

The **MF** assigns a velocity vector to each pixel in the image.

These velocities are INDUCED by the RELATIVE MOTION btw the camera and the 3D scene

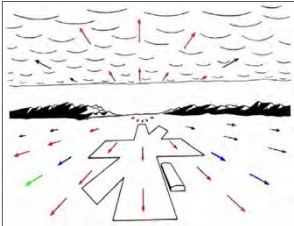
The **MF** can be thought as the projection of the 3D velocities on the image plane.

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**Motion Field**

**Non rotating Camera:**

- Vectors are radial to Focus
- **FOE:** Focus Of **Expansion**
- **FOC:** Focus Of **Contraction**
- Point where Motion vectors intersect with the image plane.



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### Motion Field

Length of vector:

- inversely proportional to distance of point
- proportional to sine of point and movement direction of camera

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### Motion Field

**Attention:** Motion Vector  $\neq$  Motion

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### Optical Flow estimation

Correspondence problem

- sparse vector field
- same problem as stereo

To reliably establish correspondences

- High temporal sampling  $\rightarrow$  small changes
- Constant intensities/edges

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### Optical Flow estimation

Two assumptions

1. Observed brightness is constant
2. Nearby points move in similar manner (velocity smoothness)

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### Brightness Constraint

Brightness Constancy Equation:

$$J(x, y) \approx I(x + u(x, y), y + v(x, y))$$

Or, minimize :

$$E(u, v) = (J(x, y) - I(x + u, y + v))^2$$

Linearizing (assuming small  $(u, v)$ ):

$$J(x, y) \approx I(x, y) + I_x(x, y) \cdot u(x, y) + I_y(x, y) \cdot v(x, y)$$

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### Gradient Constraint (or the Optical Flow Constraint)

$$E(u, v) = (I_x \cdot u + I_y \cdot v + I_t)^2$$

Minimizing:  $\frac{\partial E}{\partial u} = \frac{\partial E}{\partial v} = 0$

$$I_x(I_x u + I_y v + I_t) = 0$$

$$I_y(I_x u + I_y v + I_t) = 0$$

In general  $I_x, I_y \neq 0$

Hence,  $I_x \cdot u + I_y \cdot v + I_t \approx 0$

**The gradient constraint – only one constraint for each pixel**

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**Aperture Problem**

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**Aperture Problem**

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**Aperture Problem**

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**Motion Based Segmentation**

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**Motion Based Segmentation**

- ◆ **Top-Down:**
  - ◆ Global Motion
  - ◆ All regions that are significantly different
- ◆ **Joint estimation / segmentation**
  - ◆ Pixel as Mixture of Motions
  - ◆ Estimate Motion and Segmentation simultaneously
- ◆ **Grouping of elementary regions**
  - ◆ Region Growing Split & Merge, etc.
  - ◆ Use also Gray value information

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**Vision = Inverse Graphics**

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# Tracking = Inverse Animation

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# Tracking: First Idea!

initial position prediction measurement update

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# Region tracking

or "BLOB" tracking

recognizing a ping-pong ball by pixel intensity

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# Contour Tracking

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# Contour-tracking, applied

Automotive assembly and disassembly  
Medical image processing  
Welding  
Gesture recognition

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# Optical Tracking - Animation

multi-camera tracking system:

- real-time motion capturing of actors
- real-time visual feedback
- added redundancy by integrating single view tracking as fallback

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Tracking for MOCAP



Tracking landmarks to control avatars facial expressions (VIDEO)

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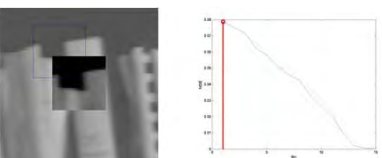
Tracking f. Augmented Reality



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Lucas-Kanade Tracking

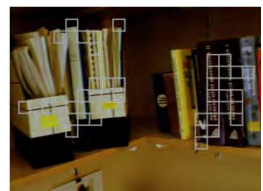
Find correspondences using a template  
 Estimate „optimal“ Translation  
 Minimize SSD  
 Gradient based Optimization



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• Computer Vision Applications:

- Optical Flow
- Feature Tracker
- KLT (Kanade-Lucas-Tomasi Tracking)
- Widely used



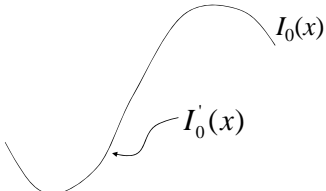
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Lucas & Kanade  
Derivation

#1

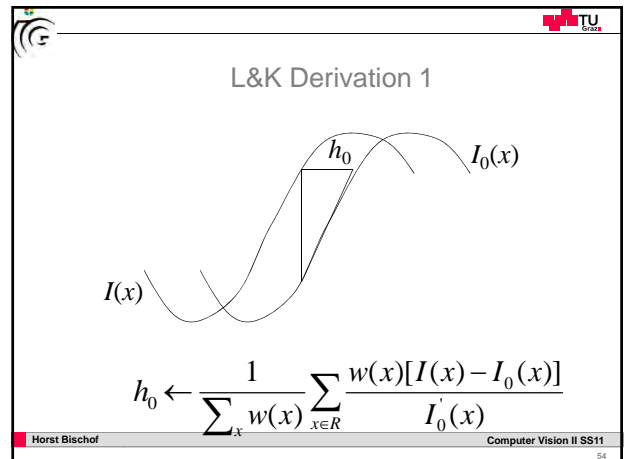
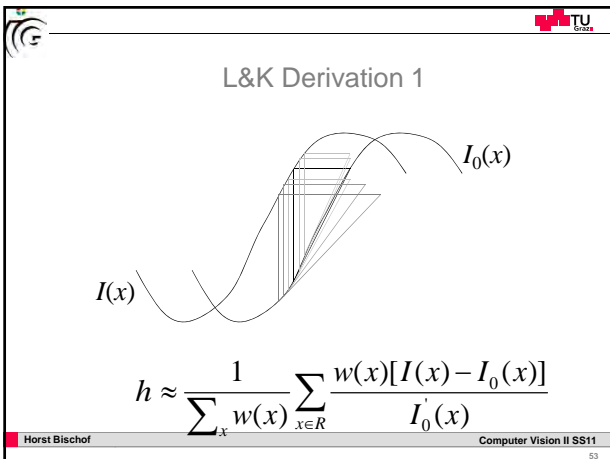
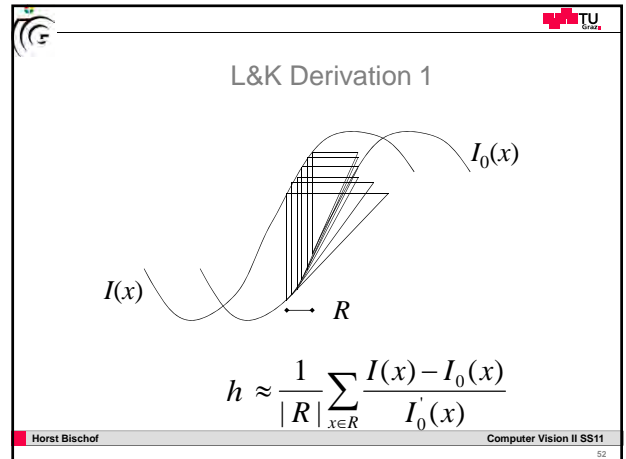
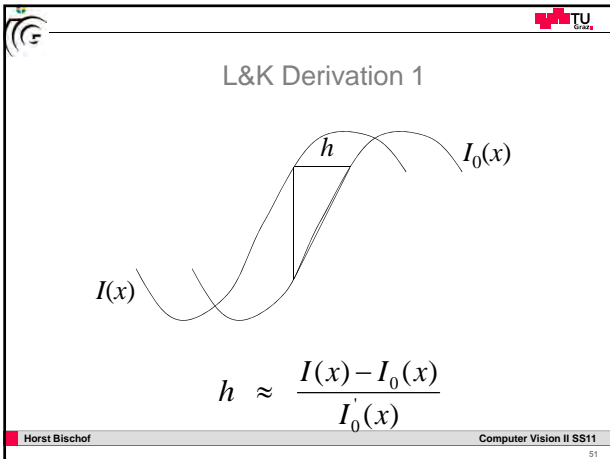
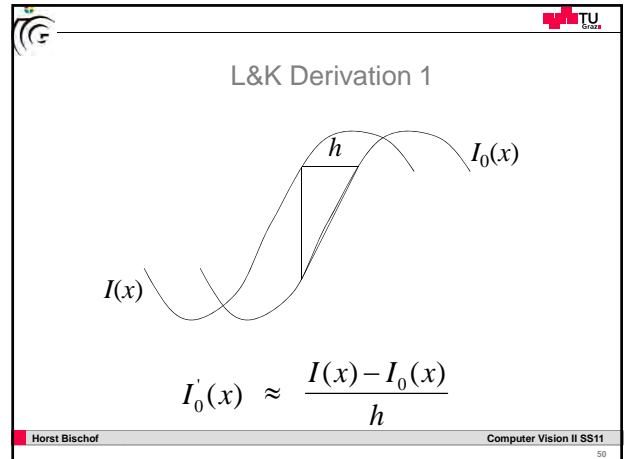
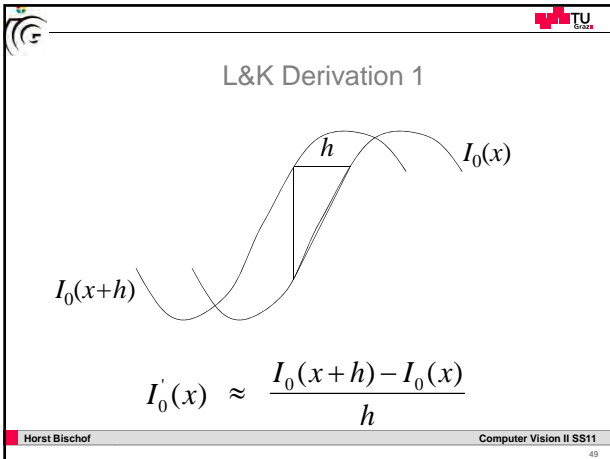
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L&K Derivation 1



$$I'_0(x) = \lim_{h \rightarrow 0} \frac{I_0(x+h) - I_0(x)}{h}$$

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L&K Derivation 1

$$h_1 \leftarrow h_0 + \frac{1}{\sum_x w(x)} \sum_{x \in R} \frac{w(x)[I(x) - I_0(x+h_0)]}{I_0'(x+h_0)}$$

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L&K Derivation 1

$$h_2 \leftarrow h_1 + \frac{1}{\sum_x w(x)} \sum_{x \in R} \frac{w(x)[I(x) - I_0(x+h_1)]}{I_0'(x+h_1)}$$

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L&K Derivation 1

$$h_{k+1} \leftarrow h_k + \frac{1}{\sum_x w(x)} \sum_{x \in R} \frac{w(x)[I(x) - I_0(x+h_k)]}{I_0'(x+h_k)}$$

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L&K Derivation 1

$$h_{k+1} \leftarrow h_k + \frac{1}{\sum_x w(x)} \sum_{x \in R} \frac{w(x)[I(x) - I_0(x+h_k)]}{I_0'(x+h_k)}$$

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Lucas & Kanade  
Derivation

#2

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L&K Derivation 2

Sum-of-squared-difference (SSD) error

$$E(h) = \sum_{x \in R} [I(x) - I_0(x+h)]^2$$

$$E(h) \approx \sum_{x \in R} [I(x) - I_0(x) - hI_0'(x)]^2$$

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L&K Derivation 2

$$\frac{\partial E}{\partial h} \approx \sum_{x \in R} 2[I_0'(x)(I(x) - I_0(x)) - hI_0''(x)^2]$$

$$= 0$$

$$h \approx \frac{\sum_{x \in R} I_0'(x)(I(x) - I_0(x))}{\sum_{x \in R} I_0''(x)^2}$$

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Comparison

$$h \approx \frac{\sum_x \frac{w(x)[I(x) - I_0(x)]}{I_0'(x)}}{\sum_x w(x)}$$

$$h \approx \frac{\sum_x I_0'(x)[I(x) - I_0(x)]}{\sum_x I_0''(x)^2}$$

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Comparison

$$h \approx \frac{\sum_x \frac{w(x) [I(x) - I_0(x)]}{I_0'(x)}}{\sum_x w(x)}$$

$$h \approx \frac{\sum_x I_0'(x)[I(x) - I_0(x)]}{\sum_x I_0''(x)^2}$$

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Generalizations

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Original

$$E(h) = \sum_{x \in R} [I(x+h) - I_0(x)]^2$$

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Original

Dimension of image

$$E(h) = \sum_{x \in R} [I(x+h) - I_0(x)]^2$$

1-dimensional

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Generalization 1a

Dimension of image

$$E(\mathbf{h}) = \sum_{\mathbf{x} \in R} [I(\mathbf{x} + \mathbf{h}) - I_0(\mathbf{x})]^2$$

2D:  $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

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Lucas-Kanade Algorithm

Iterate:

- (1) Warp  $f$  with  $W(\mathbf{x}; p)$  to compute  $f(W(\mathbf{x}; p))$
- (2) Compute the error image  $E(\mathbf{x}) = f(W(\mathbf{x}; p)) - I_0(\mathbf{x})$
- (3) Warp the gradient  $\nabla f$  with  $W(\mathbf{x}; p)$
- (4) Evaluate the Jacobian  $\frac{\partial E}{\partial p}$  at  $(\mathbf{x}, p)$
- (5) Compute the steepest descent images  $\nabla f \frac{\partial E}{\partial p}$
- (6) Compute the Hessian matrix using Equation (11)
- (7) Compute  $\sum_{\mathbf{x} \in R} \nabla f \frac{\partial E}{\partial p} \nabla f \frac{\partial E}{\partial p}^T (I_0(\mathbf{x}) - f(W(\mathbf{x}; p)))$
- (8) Compute  $\Delta p$  using Equation (10)
- (9) Update the parameters  $p \leftarrow p + \Delta p$

$p \leq \epsilon$

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Problem A

Does the iteration converge?

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Problem A

Local minima:

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Problem B

Zero gradient:

$$h \approx \frac{-\sum_{x \in R} I_0'(x)(I(x) - I_0(x))}{\sum_{x \in R} I_0'(x)^2}$$

$h$  is undefined if  $\sum_{x \in R} I_0'(x)^2$  is zero

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Problem B

Zero gradient:

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Problem B'

Aperture problem:

$$h_y \approx \frac{-\sum_{x \in R} \frac{\partial I_0(\mathbf{x})}{\partial y} (\mathbf{x})(I(\mathbf{x}) - I_0(\mathbf{x}))}{\sum_{x \in R} \left( \frac{\partial I_0(\mathbf{x})}{\partial y} \right)^2}$$

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Problem B'

No gradient along one direction:

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Good Features to Track?

- Textured Regions  
otherwise Aperture problem
- Harris corners
  - Edge: one eigenvector with eigenvalue zero
  - Homogenous: zero eigenvalues
- Other break-downs:
  - No brightness constancy
  - Nearby points move different (which window size)
  - Too much motion (multi-scale estimation)
  - Occlusions

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KLT Examples

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Solutions to A & B

Possible solutions:

- Manual intervention

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Solutions to A & B

Possible solutions:

- Manual intervention
- Zero motion default

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Solutions to A & B

Possible solutions:

- Manual intervention
- Zero motion default
- Coefficient "dampening"

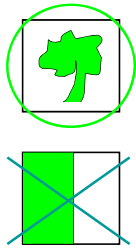
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Solutions to A & B

Possible solutions:

- Manual intervention
- Zero motion default
- Coefficient "dampening"
- Reliance on good features



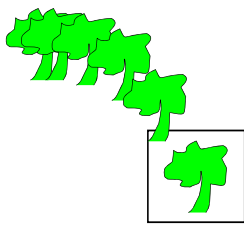
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Solutions to A & B

Possible solutions:

- Manual intervention
- Zero motion default
- Coefficient "dampening"
- Reliance on good features
- Temporal filtering



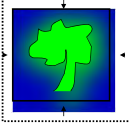
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Solutions to A & B

Possible solutions:

- Manual intervention
- Zero motion default
- Coefficient "dampening"
- Reliance on good features
- Temporal filtering
- Spatial interpolation / hierarchical estimation



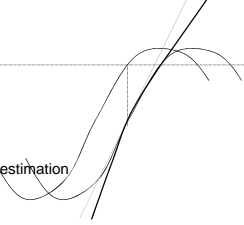
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Solutions to A & B

Possible solutions:

- Manual intervention
- Zero motion default
- Coefficient "dampening"
- Reliance on good features
- Temporal filtering
- Spatial interpolation / hierarchical estimation
- Higher-order terms

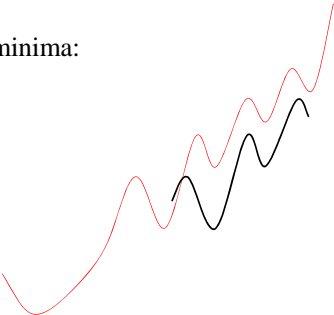


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Problem A

Local minima:



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## Summary

- Motion Detection Methods
  - Background Subtraction / Frame difference
  - Assumption and Problems
- Motion Field Estimation
  - Assumptions
  - Aperture Problem
- Tracking
  - Main Loop
- Lucas Kanade
  - Principal Idea
  - Problems, Limitations and Solution